The Electron Density Assimilative Model (EDAM)

Matthew Angling
Background

- Electron Density Assimilative Model (EDAM)
  - Data assimilation model
  - Produces 3D electron density grids
  - Developed at QinetiQ Ltd (UK) with funding from the UK MOD Science and Technology programme
    - To provide a high accuracy and timely specification of the ionosphere for use in radio systems
    - Space track radar, space based radar, single frequency GNSS, HF systems
Starting point: can be either a climatological model or a physics based model

Rigorous mathematical approach

Data Assimilation
Room temperature:
Random variable $T$, mean $<T>$, variance $\sigma^2$

Thermometer measurement, $T_o$
mean $<T>$, variance $\sigma_o^2$

Background information provided by thermostat setting, $T_b$
mean $<T>$, variance $\sigma_b^2$
Best linear unbiased estimator (BLUE)

\[ T_a = T_b + \left( \frac{\sigma_b^2}{\sigma_b^2 + \sigma_o^2} \right) \times (T_o - T_b) \]
Extension to vectors

- Variances are replaced with error covariance matrices
  - Diagonal values contain the variance of the errors
  - Off diagonal terms contain the covariances between the errors associated with different elements of the data vectors.
- Generally variables are not directly observed. Observations \( y \) are related to variables by an observation operator: \( y = Hx + \varepsilon \)

\[
\begin{align*}
x_a &= x_b + K(y - Hx_b) \\
K &= BH^T (HBH^T + R)^{-1}
\end{align*}
\]

- \( x_a \) = most probable atmospheric state
- \( x_b \) = a priori (background) atmospheric model
- \( y \) = observations
- \( B \) = background error covariance matrix
- \( H \) = Observation operator
- \( R \) = observation error covariance matrix
- \( K \) = weight matrix
Effects of varying background errors

Truth electron density

Analysis
<table>
<thead>
<tr>
<th>LT persistence forecast</th>
<th>Model</th>
<th>Representation</th>
<th>Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>LT persistence forecast</td>
<td>Empirical • IRI • RIBG • PIM</td>
<td>Shells • Single • Multiple 3D basis functions • Horiz harmonics • Vertical EOFs 3D grid • Geographic • Geomagnetic</td>
<td>Non-optimal • Profile adjustment • Tomography • ART, MART, etc</td>
</tr>
<tr>
<td>Physical forecast</td>
<td>Physical • Ionospheric • Coupled</td>
<td></td>
<td>Optimal • DIT • GMKF • Approx Kalman • Full Kalman • Variational methods</td>
</tr>
</tbody>
</table>

No covariances

No covariances
<table>
<thead>
<tr>
<th>Model</th>
<th>Representation</th>
<th>Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>LT persistence</td>
<td>Empirical</td>
<td>Non-optimal</td>
</tr>
<tr>
<td>Forecast</td>
<td>• IRI</td>
<td>• Profile adjustment</td>
</tr>
<tr>
<td></td>
<td>• RIBG</td>
<td>• Tomography</td>
</tr>
<tr>
<td></td>
<td>• PIM</td>
<td>• ART, MART, etc</td>
</tr>
<tr>
<td></td>
<td>Physical</td>
<td>Optimal</td>
</tr>
<tr>
<td></td>
<td>• Ionospheric</td>
<td>• DIT</td>
</tr>
<tr>
<td></td>
<td>• Coupled</td>
<td>• GMKF</td>
</tr>
<tr>
<td></td>
<td>Physical</td>
<td>• Approx Kalman</td>
</tr>
<tr>
<td></td>
<td>forecast</td>
<td>• Full Kalman</td>
</tr>
<tr>
<td></td>
<td>• Geographic</td>
<td>• Variational methods</td>
</tr>
<tr>
<td></td>
<td>• Geomagnetic</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3D basis functions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Horiz harmonics</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3D grid</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Vertical EOFs</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Geographic</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Geomagnetic</td>
<td></td>
</tr>
</tbody>
</table>
Electron Density Assimilative Model (EDAM)

- International Reference Ionosphere used for background model
  - Electrons only
- Designed to be scalable
  - Can assimilate single or multiple measurements
- Low demands on computer resources
- Simple evolution
  - Exponential decay of electron density grid differences
- Uses sun-fixed geomagnetic coordinate system
- Model Variances are propagated, covariance are estimated as required
Data assimilation allows the use of a wide range of data types:
- Vertical, oblique and backscatter ionosondes
- In-situ measurements of electron density
- Extreme ultra-violet measurements
- Total electron content from GPS
- Ground or space based
Digisonde and GPS locations
October 2003 – background model
October 2003 - EDAM
Testing methods

- Against independent data
  - Vertical ionograms, radio occultation data etc
- Against independent models
  - GAIM (Utah State University)
  - GPSII (NWRA)
  - TIE-CGM (NCAR)
  - GTIM (U of Michigan)

EDAM in ESPAS
EDAM in ESPAS
EDAM in ESPAS – map output

QinetiQ

EDAM v1.2.07 NmF2 30/09/2015 11:45:00 UT

UNIVERSITY OF BIRMINGHAM

Generated by EMMENTAL on 30/09/2015 12:50:31 UT
<table>
<thead>
<tr>
<th>EONEX VERSION / TYPE</th>
<th>PGM / RUN BY / DATE</th>
<th>DESCRIPTION</th>
<th>DESCRIPTION</th>
<th>DESCRIPTION</th>
<th>COMMENT</th>
<th>EPOCH OF FIRST MAP</th>
<th>EPOCH OF LAST MAP</th>
<th>INTERVAL</th>
<th># OF MAPS IN FILE</th>
<th>ELEVATION CUTOFF</th>
<th>OBSERVABLES CUTOFF</th>
<th># OF STATIONS</th>
<th># OF SATELLITES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data stored as log(Ne)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>9</td>
<td>30</td>
<td>11</td>
<td>45</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>9</td>
<td>30</td>
<td>11</td>
<td>45</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One-way carrier phase leveled to code</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6371.0</td>
<td>3</td>
<td>100</td>
<td>46</td>
<td>90</td>
<td>-1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>90.00</td>
<td>95.00</td>
<td>100.00</td>
<td>105.00</td>
<td>110.00</td>
<td>115.00</td>
<td>120.00</td>
<td>125.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>130.00</td>
<td>135.00</td>
<td>140.00</td>
<td>150.00</td>
<td>160.00</td>
<td>170.00</td>
<td>180.00</td>
<td>190.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200.00</td>
<td>210.00</td>
<td>220.00</td>
<td>230.00</td>
<td>240.00</td>
<td>250.00</td>
<td>260.00</td>
<td>270.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>280.00</td>
<td>290.00</td>
<td>300.00</td>
<td>310.00</td>
<td>320.00</td>
<td>330.00</td>
<td>340.00</td>
<td>350.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>360.00</td>
<td>370.00</td>
<td>380.00</td>
<td>390.00</td>
<td>400.00</td>
<td>410.00</td>
<td>420.00</td>
<td>430.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>440.00</td>
<td>460.00</td>
<td>480.00</td>
<td>500.00</td>
<td>520.00</td>
<td>540.00</td>
<td>560.00</td>
<td>580.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>600.00</td>
<td>620.00</td>
<td>640.00</td>
<td>660.00</td>
<td>680.00</td>
<td>700.00</td>
<td>720.00</td>
<td>740.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>760.00</td>
<td>780.00</td>
<td>800.00</td>
<td>820.00</td>
<td>840.00</td>
<td>860.00</td>
<td>880.00</td>
<td>900.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>920.00</td>
<td>940.00</td>
<td>960.00</td>
<td>980.00</td>
<td>1000.00</td>
<td>1020.00</td>
<td>1040.00</td>
<td>1060.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Metres Above Sea Level</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.00  95.00  100.00  105.00  110.00  115.00  120.00  125.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>130.00  135.00  140.00  150.00  160.00  170.00  180.00  190.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200.00  210.00  220.00  230.00  240.00  250.00  260.00  270.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>280.00  290.00  300.00  310.00  320.00  330.00  340.00  350.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>360.00  370.00  380.00  390.00  400.00  410.00  420.00  430.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>440.00  460.00  480.00  500.00  520.00  540.00  560.00  580.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>600.00  620.00  640.00  660.00  680.00  700.00  720.00  740.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>760.00  780.00  800.00  820.00  840.00  860.00  880.00  900.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>920.00  940.00  960.00  980.00  1000.00  1020.00  1040.00  1060.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1080.00  1100.00  1120.00  1140.00  1160.00  1180.00  1200.00  1220.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1240.00  1260.00  1280.00  1300.00  1320.00  1340.00  1360.00  1380.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1400.00  1420.00  1440.00  1460.00  1480.00  1500.00  1520.00  1540.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1560.00  1580.00  1600.00  1620.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**END OF HEIGHT GRID**

<table>
<thead>
<tr>
<th>Degrees North of Prime Meridian</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
</tr>
<tr>
<td>46</td>
</tr>
<tr>
<td>-90.00 -86.00 -82.00 -78.00 -74.00 -70.00 -66.00 -62.00 -58.00 -54.00 -50.00</td>
</tr>
<tr>
<td>-46.00 -42.00 -38.00 -34.00 -30.00 -26.00 -22.00 -18.00 -14.00 -10.00 -6.00</td>
</tr>
<tr>
<td>-2.00  2.00  6.00  10.00  14.00  18.00  22.00  26.00  30.00  34.00  38.00</td>
</tr>
<tr>
<td>42.00  46.00  50.00  54.00  58.00  62.00  66.00  70.00  74.00  78.00  82.00</td>
</tr>
<tr>
<td>86.00  90.00</td>
</tr>
<tr>
<td>46</td>
</tr>
<tr>
<td>90</td>
</tr>
</tbody>
</table>

**END OF LAT GRID**

<table>
<thead>
<tr>
<th>Degrees East of Greenwich</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
</tr>
<tr>
<td>44.00  48.00  52.00  56.00  60.00  64.00  68.00  72.00  76.00  80.00  84.00</td>
</tr>
<tr>
<td>88.00  92.00  96.00  100.00  104.00  108.00  112.00  116.00  120.00  124.00  128.00</td>
</tr>
<tr>
<td>132.00  136.00  140.00  144.00  148.00  152.00  156.00  160.00  164.00  168.00  172.00</td>
</tr>
<tr>
<td>176.00  180.00  184.00  188.00  192.00  196.00  200.00  204.00  208.00  212.00  216.00</td>
</tr>
<tr>
<td>220.00  224.00  228.00  232.00  236.00  240.00  244.00  248.00  252.00  256.00  260.00</td>
</tr>
<tr>
<td>264.00  268.00  272.00  276.00  280.00  284.00  288.00  292.00  296.00  300.00  304.00</td>
</tr>
<tr>
<td>308.00  312.00  316.00  320.00  324.00  328.00  332.00  336.00  340.00  344.00  348.00</td>
</tr>
<tr>
<td>352.00  356.00</td>
</tr>
</tbody>
</table>

**END OF LON GRID**

Length: 3324892 lines: 45601 Ln: 14 Col: 37 Sel: 0 | 0 Dos/Windows UTF-8 w/o BOM INS
<table>
<thead>
<tr>
<th>Year</th>
<th>Month</th>
<th>Day</th>
<th>Hour</th>
<th>Minute</th>
<th>Second</th>
<th>Latitude (Degrees)</th>
<th>Longitude (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015</td>
<td>9</td>
<td>30</td>
<td>11</td>
<td>45</td>
<td>0</td>
<td>8.673</td>
<td>9.525</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.163</td>
<td>10.407</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.386</td>
<td>10.293</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.114</td>
<td>9.925</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9.763</td>
<td>9.679</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9.672</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-90.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>10</td>
<td>20</td>
<td>12</td>
<td>45</td>
<td>0</td>
<td>9.829</td>
<td>10.064</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.252</td>
<td>10.443</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.624</td>
<td>10.797</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.965</td>
<td>11.112</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11.231</td>
<td>11.321</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11.394</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-90.0</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2017</td>
<td>11</td>
<td>30</td>
<td>11</td>
<td>45</td>
<td>0</td>
<td>11.442</td>
<td>11.472</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11.483</td>
<td>11.483</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11.474</td>
<td>11.463</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11.447</td>
<td>11.429</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11.406</td>
<td>11.381</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11.353</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-90.0</td>
<td>8.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2018</td>
<td>12</td>
<td>31</td>
<td>12</td>
<td>45</td>
<td>0</td>
<td>10.884</td>
<td>10.829</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.777</td>
<td>10.777</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.679</td>
<td>10.634</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.592</td>
<td>10.551</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.512</td>
<td>10.475</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.437</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-90.0</td>
<td>12.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Conclusions

- Electron Density Assimilative Model (EDAM)
  - Data assimilation model developed at QinetiQ in the UK
  - Runs in near real time
  - Routinely assimilates ionosondes and GPS-TEC
  - Provides 3D grids into ESPAS